SD575 Midterm Examination
October 19, 2006 9:30am – 11:30am
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1. [15 marks] For the indicated frequency response, answer the following questions.

$$H(u) = -4\pi^2 u^2 e^{-\left(\frac{u}{a}\right)^2}$$

(a) Sketch | H(u) | and show at least |H(0)|, u_{max}, and $|H(u_{max})|$.
(b) Calculate u_{max}, the value for u where the maximum for H(u) occurs.
(c) What type of a filter is this? Why?
(d) The filter H(u) can be implemented as two consecutive filters, $H_1(u)$ and $H_2(u)$. Indicate appropriate functions for $H_1(u)$ and $H_2(u)$. Describe their operations in the spatial domain either through their impulse responses or with their effect on an input f(x). Is there any implementation constraint that might affect the order in which the two operations should be applied?
(e) Determine and sketch the impulse response associated with H(u).
(f) If an input of the form f(x) is applied, find and sketch the output.

$$f(x) = \begin{cases} 0, & x \le 0 \\ x, & 0 < x \le \dfrac{1}{\pi a} \\ \dfrac{1}{\pi a}, & x > \dfrac{1}{\pi a} \end{cases}$$

2. [10 marks] (a) Calculate and sketch the step response $g_e(x)$ to an exponential low pass filter $h_e(x) = \frac{a}{2} e^{-a|x|}$.
(b) The edge strength can be considered as a measure of slope at the center of the edge (the higher the slope, the higher the edge strength). Using this knowledge, what should the parameter 'a' be set to so that the exponential low pass filter would have the equivalent edge strength as a Gaussian low pass filter $h_G(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-0.5\left(\frac{x}{\sigma}\right)^2}$?
(c) What is a shortcoming of the use of the edge strength operator described in (b) for application to digital image processing?

3. [10 marks] Point operations can be used to modify the appearance of an image. Here, the input image is defined by f(m,n) = r and g(m,n) = s = T(r) where T(r) = ar + b. Assume that each of these is normalized, $0 \le r \le 1$ and $0 \le s \le 1$.
(a) Find a and b in terms of m_r and σ_r^2, the mean and variance of *r*, if you want $m_s = 0.5$ and $\sigma_s^2 = 0.25\ \sigma_r^2$.
(b) Suppose that you want to reduce the global dynamic range (the low frequency variation across the whole image) while enhancing local contrast (high frequency, edge components). More specifically, increase the edge variance by a factor of 4 and decrease the low frequency variance by a factor of 4. Explain your strategy and the resulting system.
(c) What impact will your overall system in (b) have on the dynamic range of the output image? If there is any negative impact, then explain what can be done to remedy the problem.