SYDE 575: Introduction to Image Processing

Image Restoration in Frequency Domain
Part II

Image Degradation

- Image degradation model

$$g(x, y) = \underbrace{H}_{\text{Degradation function}}\left[f(x, y) \right] + \underbrace{\eta}_{\text{Noise}}(x, y)$$

- Many types of degradation can be approximated by linear, position-invariant processes

Linear, Position-Invariant Degradations

- *H* is linear if it obeys the following properties

- Additivity

$$H\left[f_1(x,y)+f_2(x,y)\right] = H\left[f_1(x,y)\right]+H\left[f_2(x,y)\right]$$

- Homogeneity

$$H\left[af_1(x,y)\right] = aH\left[f_1(x,y)\right]$$

Linear, Position-Invariant Degradations

- *H* is position-invariant if

$$H\left[f(x-\alpha, y-\beta)\right] = g(x-\alpha, y-\beta)$$

- Response at any pixel in image depends only on value of input at the pixel, not its position

Linear, Position-Invariant Degradations

- In the absence of additive noise

$$g(x,y) = H[f(x,y)] = H\left[\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(\alpha,\beta)\delta(x-\alpha, y-\beta)\,d\alpha\, d\beta\right]$$

- If H is linear (additivity and homogeneity)

$$g(x,y) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(\alpha,\beta)H\left[\delta(x-\alpha, y-\beta)\right]d\alpha\, d\beta$$

Linear, Position-Invariant Degradations

- If H is position invariant

$$H\left[\delta(x-\alpha, y-\beta)\right] = h(x-\alpha, y-\beta)$$

- Therefore

$$g(x,y) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(\alpha,\beta)h(x-\alpha, y-\beta)d\alpha\, d\beta$$

- Just convolution!

Image Restoration

- A linear, position-invariant degradation can be represented by a convolution of the input image and additive noise
- Intuitively, the goal of image restoration in such a case is to find filters that reverse the original degradation filtering process
- Commonly called “deconvolution”

Estimating Degradation Function

- Estimation by Image Observation
  - Take subimage with simple structures from image
  - Construct estimate of what subimage should be like prior to degradation
  - Determine subimage degradation function h_s based on observed subimage g_s and constructed subimage f_s

$$H_s(u,v) = \frac{G_s(u,v)}{\hat{F}_s(u,v)}$$

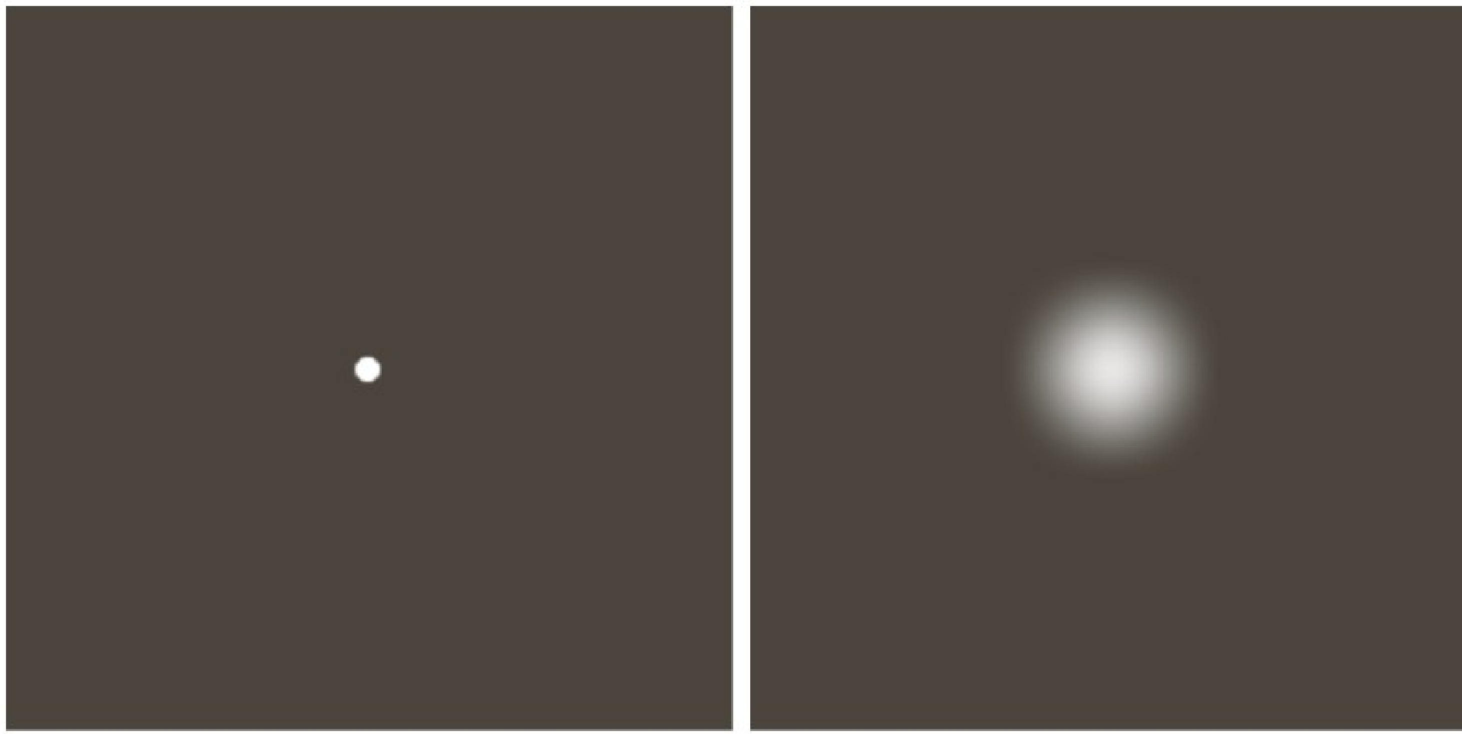
Estimating Degradation Function

- Estimation by Image Observation
  - Reconstruct complete degradation function h based on h_s
  - Works based on the assumption of position invariance

Estimating Degradation Function

- Estimation by Experimentation
  - Useful if equipment similar to equipment used to acquire degraded image available
  - Image a impulse (small dot of light) and adjust settings until the impulse is close to that produced by the degradation
  - Use the estimated degradation function to restore image

Estimation by Experimentation



a b

FIGURE 5.24 Degradation estimation by impulse characterization. (a) An impulse of light (shown magnified). (b) Imaged (degraded) impulse.

Source: Gonzalez and Woods

Estimating Degradation Function

- Estimation by Modelling
  - Useful if physical conditions (camera conditions, environment conditions, etc.) can be modelled
  - Example: Linear motion blur

$$h(x, y) = \frac{1}{L}\delta\,(\vec{L})$$

- e.g., for L=4 and horizontal motion blur
  - h=[0.1111   0.1111   0.1111   0.1111]

Estimation by Modelling

L=9, horizontal motion blur

Inverse Filtering

- Simple idea:
  - Degradation assumed to be created by multiplying degradation function by image in frequency domain
  - Removing degradation would mean dividing degraded image by degradation function

$$\hat{F}(u,v) = \frac{G(u,v)}{H(u,v)}$$

Inverse Filtering

- Problem: What happens when there is noise?

$$\hat{F}(u,v) = F(u,v) + \frac{N(u,v)}{H(u,v)}$$

- If degradation has very small values, N/H dominates F!
- Frequently worse than original degradation image!

Inverse Filtering

a b
c d

FIGURE 5.27 Restoring Fig. 5.25(b) with Eq. (5.7-1). (a) Result of using the full filter. (b) Result with *H* cut off outside a radius of 40; (c) outside a radius of 70; and (d) outside a radius of 85.

Source: Gonzalez and Woods

Wiener (Least Squares) Filtering

- Recall:

$$\underset{\substack{\uparrow \\ \text{Sensor measurement}}}{\underline{m}} = \underset{\substack{\uparrow \\ \text{scene}}}{\underline{z}} + \underset{\substack{\uparrow \\ \text{noise}}}{\underline{v}}$$

- Estimate scene (*z*) by minimizing least squares error

$$\underset{\substack{\uparrow \\ \text{estimate}}}{\underline{\hat{z}}} = \arg\min\left(E\left[\left(\underline{\hat{z}} - \underline{z} \right)^T \left(\underline{\hat{z}} - \underline{z} \right) \right] \right)$$

Wiener (Least Squares) Filtering

- Assuming that noise and image are uncorrelated and that one or the other has zero mean:

$$\underset{\text{estimate}}{\hat{Z}} = (\underset{\text{degradation}}{H}^T \underset{\substack{\text{Noise power} \\ \text{spectrum}}}{S_n}^{-1} H + \underset{\substack{\text{Image} \\ \text{power} \\ \text{spectrum}}}{S_f}^{-1})^{-1} H^T S_n^{-1} \underset{\text{measurement}}{M}$$

- Note: variance in spatial domain becomes power spectrum in frequency domain

Wiener (Least Squares) Filtering

- Expressing in terms of degraded image

$$\hat{F} = \left(\frac{|H|^2}{S_n} + \frac{1}{S_f}\right)^{-1} \frac{H^*}{S_n} G$$

$$\hat{F} = \left[\frac{H^* S_f}{|H|^2 S_f + S_n}\right] G$$

$$\hat{F} = \left[\frac{1}{H} \frac{|H|^2}{|H|^2 + S_n / S_f}\right] G$$

- H^*: complex conjugate of H
- $S_n = |N|^2$
- $S_f = |F|^2$

Wiener (Least Squares) Filtering

- Power spectrum of undegraded image often not known
- Solution: Replace ratio between noise and image variance by specified constant K

$$\hat{F} = \left[\frac{1}{H} \frac{|H|^2}{|H|^2 + K} \right] G$$

Wiener Filtering

FIGURE 5.29 (a) 8-bit image corrupted by motion blur and additive noise. (b) Result of inverse filtering. (c) Result of Wiener filtering. (d)–(f) Same sequence, but with noise variance one order of magnitude less. (g)–(i) Same sequence, but noise variance reduced by five orders of magnitude from (a). Note in (h) how the deblurred image is quite visible through a "curtain" of noise.

Source: Gonzalez and Woods