SYDE 575: Introduction to Image Processing

Adaptive Spatial Filters for Noise Reduction

Recap: Spatial smoothing filters

- All of the filters discussed so far give no regard to the underlying image characteristics
- This is problematic since images are generally non-stationary in nature (e.g., the image characteristics vary from one point to another)
- This results in
  - Oversmoothing of visually important detail (e.g., edges)
  - Undersmoothing of noise in smooth regions in high noise cases

Adaptive spatial smoothing filters

- Solution: change the amount of spatial smoothing based on the underlying characteristics of the image
- What do we want to do?
  - Preserve edges and other image detail
    - Reduce smoothing where such detail exists
  - Reduce noise that are very visible to the observer
    - Increase smoothing where smooth regions with little detail exists
- How do we do that?
  - Statistical measures (e.g., mean and variance) give a good indication of image characteristics

Bayesian Least Squares

- Model

$$\underset{\substack{\uparrow \\ \text{Sensor} \\ \text{measurement}}}{\underline{m}} = \underset{\substack{\uparrow \\ \text{scene}}}{\underline{z}} + \underset{\substack{\uparrow \\ \text{noise}}}{\underline{v}}$$

- Estimate scene (*z)* by minimizing least squares error

$$\underset{\substack{\uparrow \\ \text{estimate}}}{\underline{\hat{z}}} = \arg\min\left(E\left[\left(\underline{\hat{z}} - \underline{z} \right)^{T} \left(\underline{\hat{z}} - \underline{z} \right) \right] \right)$$

Bayesian Least Squares

- Bayesian least squares estimate is

$$\underset{\substack{\uparrow\\ \text{estimate}}}{\underline{\hat{z}}} = \underset{\substack{\uparrow\\ \text{mean}}}{\underline{\mu}} + \left(\underset{\substack{\uparrow\\ \text{Noise}\\ \text{covariance}}}{R}^{-1} + \underset{\substack{\uparrow\\ \text{Process}\\ \text{covariance}}}{P}^{-1} \right)^{-1} R^{-1} \left(\underset{\substack{\uparrow\\ \text{measurement}}}{\underline{m}} - \underline{\mu} \right)$$

Bayesian Least Squares

- How does that translate to images?

$$\underset{\substack{\uparrow\\ \text{estimate}}}{\hat{f}} = \mu_g + \left(\frac{1}{\underset{\substack{\uparrow\\ \text{Noise}\\ \text{variance}}}{\sigma_n^2}} + \frac{1}{\underset{\substack{\uparrow\\ \text{Image}\\ \text{variance}}}{\sigma_g^2}} \right)^{-1} \frac{1}{\sigma_n^2} \left(g - \mu_g \right)$$

Bayesian Least Squares

- Re-arranging the Bayesian Least Squares estimator for image yields

$$\hat{f} = \left(\frac{\sigma_g^2}{\sigma_g^2 + \sigma_n^2} \right) g + \left(1 - \frac{\sigma_g^2}{\sigma_g^2 + \sigma_n^2} \right) \mu_g$$

- Also call Lee's local statistics filter

Bayesian Least Squares

- Observations
  - As local image variance increases gets higher relative to noise variance, filter returns value close to *g*.
    - High local image variance associated with edges and image detail, which need to be preserved
  - As local image variance decreases, filter returns value close to the mean of pixels.
    - Low local image variance is associated with smooth regions, which needs to be smoothed to remove noise
- These observations meet our requirements!

Example

Gaussian spatial filter

Lee filter