Image Enhancement by Point Processing

Chapter 3
Point Processing

- Simplest of image enhancement techniques
- Processing based only on intensity of individual pixels

\[ s = T(z) \]

output pixel \quad transform \quad input pixel
Intensity Transformation Function

\[ s = T(z) \]

Source: Gonzalez and Woods
Example: Negative
FIGURE 2.41
Images exhibiting
(a) low contrast,
(b) medium contrast, and
(c) high contrast.

Source: Gonzalez and Woods
Gamma Correction

- Various devices used for image acquisition and display respond based on power law.
- Process used to correct power law response phenomena is called *gamma correction*.

\[ S = z^\gamma \]

- \( S \) = output pixel
- \( z \) = input pixel
- \( \gamma \) = gamma
Gamma Curves

Source: Gonzalez and Woods
Gamma Correction: Example

Source: Gonzalez and Woods
Power-law Transformation for Contrast Enhancement

- May also be used to adjust contrast of an image by either expanding or compression gray levels

\[ \gamma < 1, \text{ gray-level expansion} \]
\[ \gamma > 1, \text{ gray-level compression} \]
Gray-level expansion

FIGURE 3.8
(a) Magnetic resonance image (MRI) of a fractured human spine.
(b)–(d) Results of applying the transformation in Eq. (3.2-3) with $c = 1$ and
$\gamma = 0.6, 0.4, and 0.3$, respectively.
(Original image courtesy of Dr. David R. Pickens, Department of Radiology and Radiological Sciences, Vanderbilt University Medical Center.)

Source: Gonzalez and Woods
Gray-level Compression

FIGURE 3.9
(a) Aerial image. (b)–(d) Results of applying the transformation in Eq. (3.2-3) with $c = 1$ and $\gamma = 3.0$, 4.0, and 5.0, respectively. (Original image for this example courtesy of NASA.)

Source: Gonzalez and Woods
Histograms

- **Histogram**
  - Discrete function $h(z_k)$ showing the number of occurrences $n_k$ for the $k^{th}$ gray level $z_k$

  $$h(z_k) = n_k$$

- **Normalized Histogram**
  - Gives estimate of probability of occurrence of gray level $z_k$ *(probability distribution function)*

  $$p(z_k) = \frac{n_k}{n}, \quad \sum_{0}^{L-1} p(z_k) = 1$$
Histograms: Example

Source: Gonzalez and Woods
High-contrast images are often desirable from a visual perspective. High-contrast images have histograms where the components cover a wide dynamic range. Distribution of pixels are close to a uniform distribution. Intuitively, low-contrast images can be enhanced by transforming its pixel distribution into a uniform distribution to achieve high contrast.
Histogram Equalization

- Let $p(z)$ and $p(s)$ denote the probability density functions of $z$ and $s$
- For $s=T(z)$, $p(s)$ can be expressed as,

$$p(s) = p(z) \left| \frac{dz}{ds} \right|$$

- Since we want to transform the input image such that the pixel distribution is uniform,

$$p(s) = 1, \quad 0 \leq s \leq 1$$
Histogram Equalization

- What transformation will give you a uniform distribution for \( s \)?
  - Cumulative distribution function (CDF)

\[
s = T(z) = \int_0^z p(w) \, dw
\]

- Why?

\[
\frac{ds}{dz} = \frac{d}{dz} \left[ \int_0^z p(w) \, dw \right] = p(z) \quad \Rightarrow \quad p(s) = p(z) \left| \frac{1}{p(z)} \right| = 1
\]
FIGURE 3.18 (a) An arbitrary PDF. (b) Result of applying the transformation in Eq. (3.3-4) to all intensity levels, \( r \). The resulting intensities, \( s \), have a uniform PDF, independently of the form of the PDF of the \( r \)'s.

Source: Gonzalez and Woods
Histogram Equalization

- For discrete case,

\[ s_k = T(z_k) = \sum_{j=0}^{k} p(z_j) \]

\[ p(z_k) = \frac{n_k}{n}, \sum_{0}^{L-1} p(z_k) = 1 \]
Histogram Equalization: Example

<table>
<thead>
<tr>
<th>$r_k$</th>
<th>$n_k$</th>
<th>$p_r(r_k) = n_k/MN$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$r_0 = 0$</td>
<td>790</td>
<td>0.19</td>
</tr>
<tr>
<td>$r_1 = 1$</td>
<td>1023</td>
<td>0.25</td>
</tr>
<tr>
<td>$r_2 = 2$</td>
<td>850</td>
<td>0.21</td>
</tr>
<tr>
<td>$r_3 = 3$</td>
<td>656</td>
<td>0.16</td>
</tr>
<tr>
<td>$r_4 = 4$</td>
<td>329</td>
<td>0.08</td>
</tr>
<tr>
<td>$r_5 = 5$</td>
<td>245</td>
<td>0.06</td>
</tr>
<tr>
<td>$r_6 = 6$</td>
<td>122</td>
<td>0.03</td>
</tr>
<tr>
<td>$r_7 = 7$</td>
<td>81</td>
<td>0.02</td>
</tr>
</tbody>
</table>

**TABLE 3.1**

Intensity distribution and histogram values for a 3-bit, 64 × 64 digital image.

Source: Gonzalez and Woods
Histogram Equalization: Example

**Figure 3.19** Illustration of histogram equalization of a 3-bit (8 intensity levels) image. (a) Original histogram. (b) Transformation function. (c) Equalized histogram.

Source: Gonzalez and Woods
Histogram Equalization: Example

FIGURE 3.20 Left column: images from Fig. 3.16. Center column: corresponding histogram-equalized images. Right column: histograms of the images in the center column.

Source: Gonzalez and Woods
Local Histogram Equalization

- Global approach good for overall contrast enhancement
- However, there may be cases where it is necessary to enhance details over small areas in image
- Solution: perform histogram equalization over a small neighborhood
Local Histogram Equalization: Example

**FIGURE 3.26** (a) Original image. (b) Result of global histogram equalization. (c) Result of local histogram equalization applied to (a), using a neighborhood of size $3 \times 3$.

Source: Gonzalez and Woods